

Magnetic Field Induced Conductivity of the Vacuum of Gluodynamics

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T.K. Kalaydzhyan, E.V.Luschevskaya, M.I. Polikarpov*

arXiv:1003.2180, arXiv:0910.4682, arXiv:0909.2350,
arXiv:0909.1808, arXiv:0907.0494, arXiv:0906.0488,
arXiv:0812.1740

P- and CP-odd Effects in Hot and Dense Matter

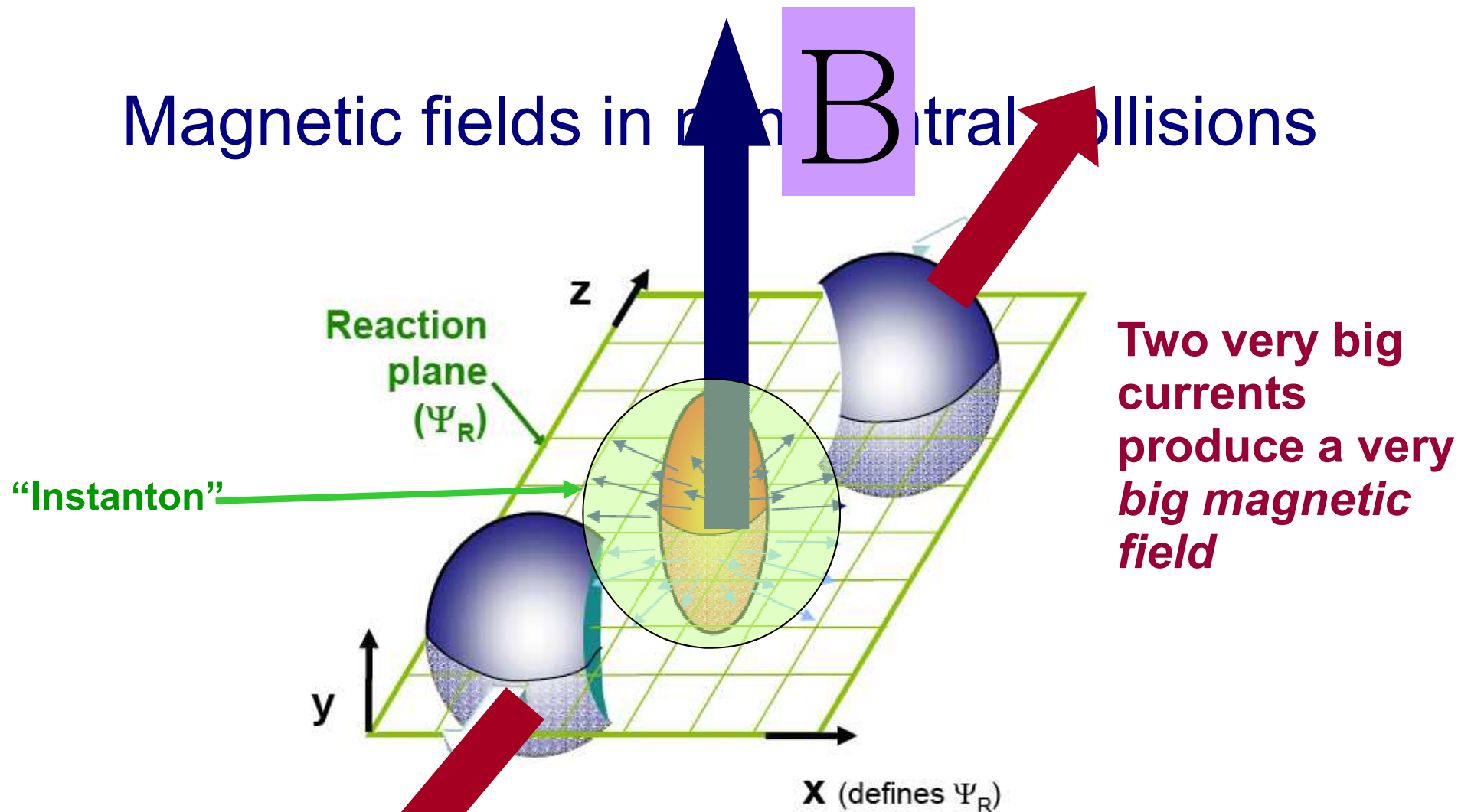
RIKEN BNL Research Center Workshop
April 26-30, 2010 at Brookhaven National Laboratory



Lattice simulations with magnetic fields

- 1. Magnetic Field Induced Conductivity [generation of the electric current of quarks along the magnetic field]**
- 2. Chiral symmetry breaking**
- 3. Magnetization of the vacuum**
- 4. Electric dipole moment of quark along the direction of the magnetic field**
- 5. Quark mass dependence of CME**

Magnetic fields in non-central collisions

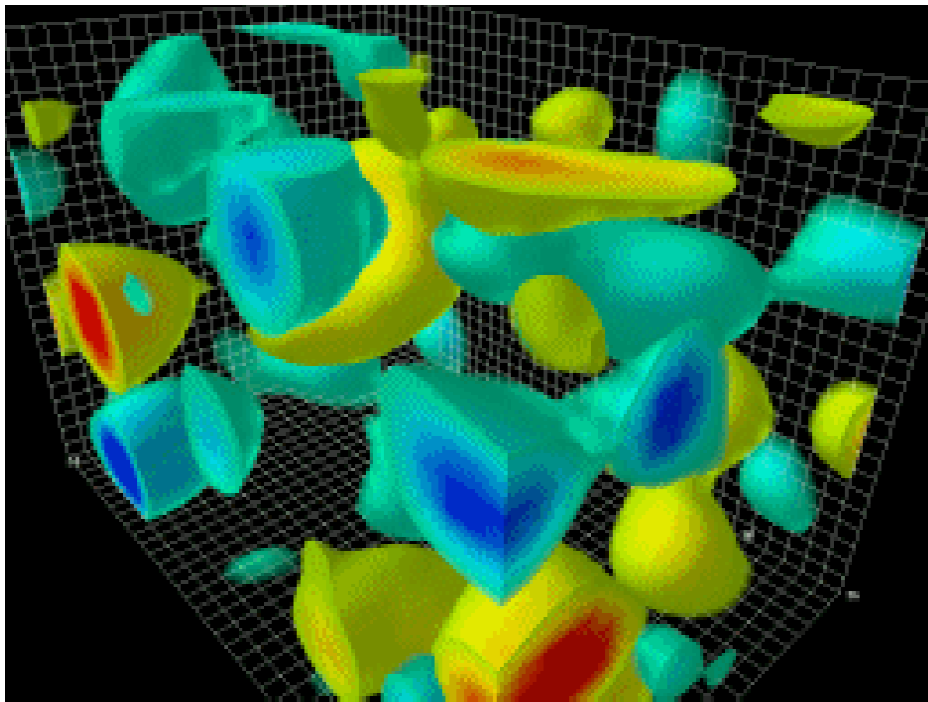


The medium is filled by electrically charged particles

Large orbital momentum, perpendicular to the reaction plane

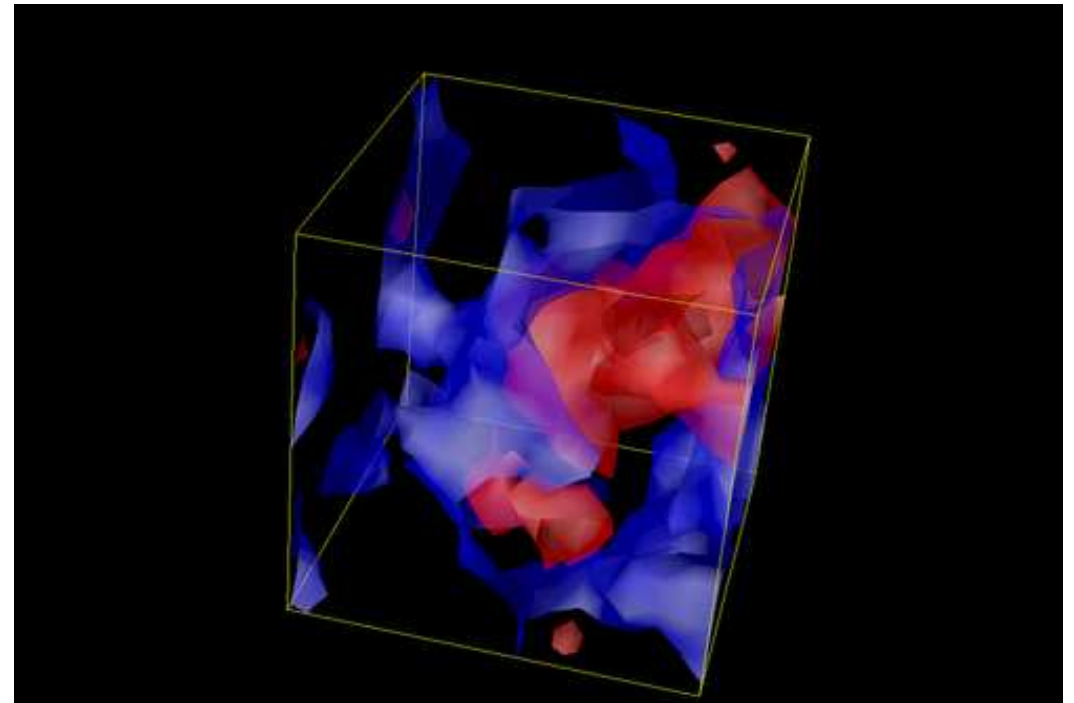
Large magnetic field along the direction of the orbital momentum

3D time slices of topological charge density, lattice calculations



D. Leinweber

Topological charge density after
vacuum cooling



P.V.Buividovich,
T.K. Kalaydzhyan, M.I. Polikarpov

Fractal topological charge density
without vacuum cooling

**We consider very strong magnetic
fields,
Magnetic forces are of the order of
strong interaction forces**

$$eB \approx \Lambda_{QCD}^2$$

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**We expect the influence of magnetic field on
strong interaction physics**

**Magnetic forces are of the order of
strong interaction forces**

$$eB \approx \Lambda_{QCD}^2$$

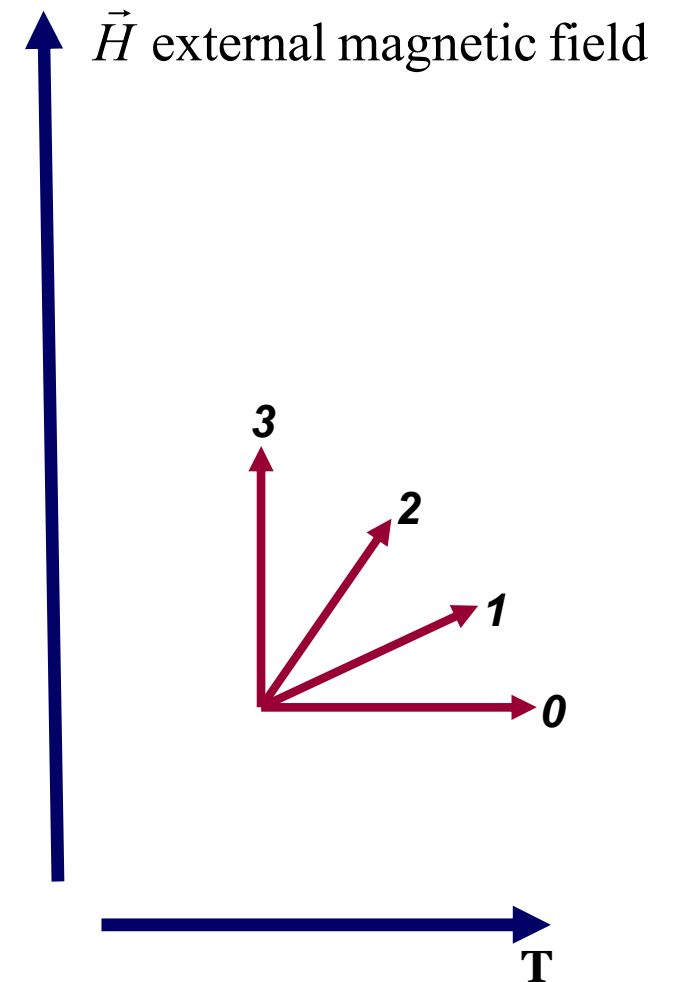
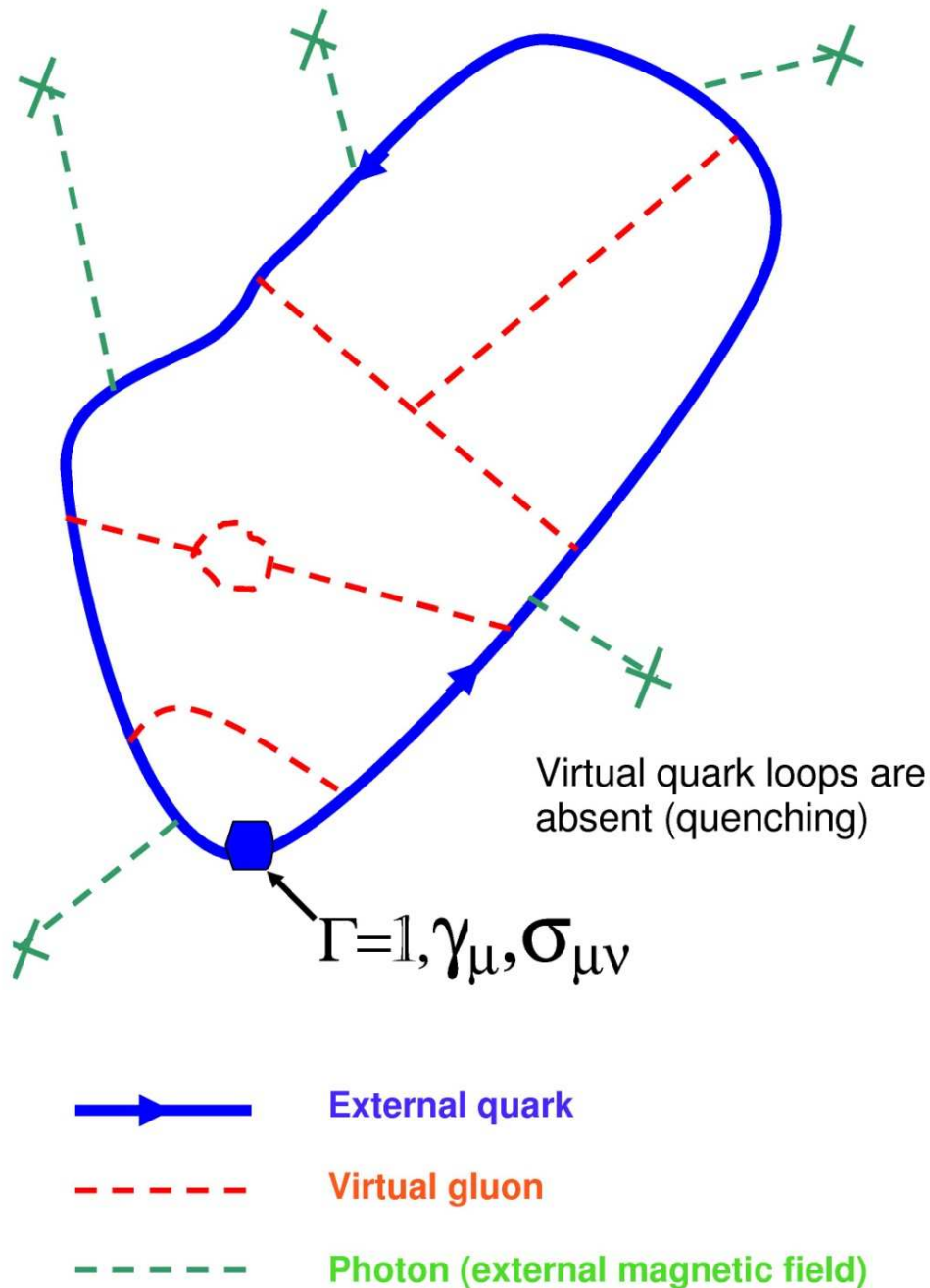
**We expect the influence of magnetic field on
strong interaction physics**

The effects are nonperturbative,

and we use

Lattice Calculations

We calculate $\langle \bar{\psi} \Gamma \psi \rangle$; $\Gamma = 1, \gamma_\mu, \sigma_{\mu\nu}$
 in the external magnetic field and in the
 presence of the vacuum gluon fields We
 consider SU(2) gauge fields and
 quenching approximation

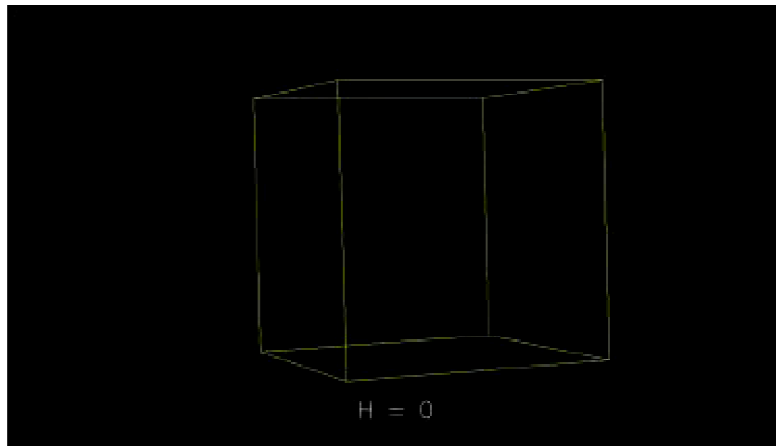


Quenched vacuum, overlap Dirac operator, external magnetic field

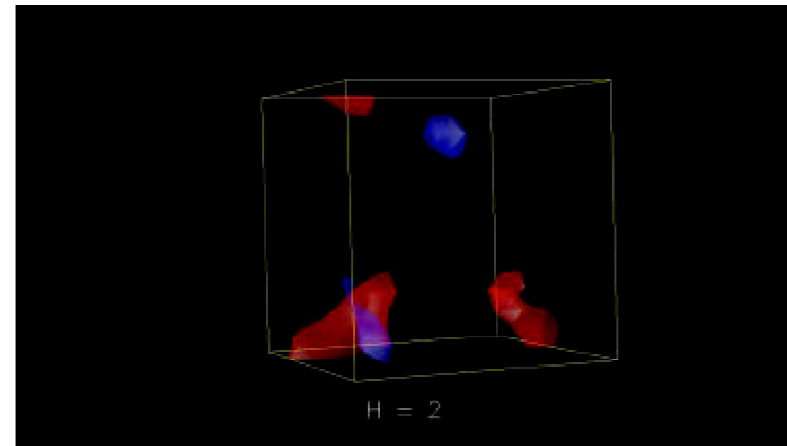
$$eB = \frac{2\pi qk}{L^2}; eB \geq 250 \text{ Mev}$$

Density of the electric charge vs. magnetic field, 3D time slices

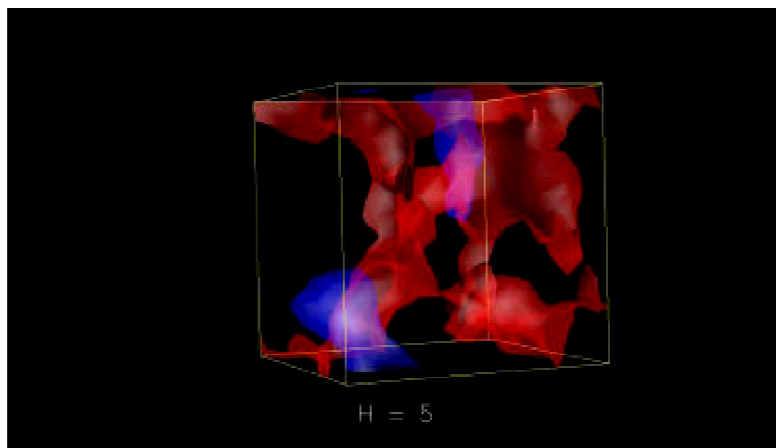
$$B = 0$$



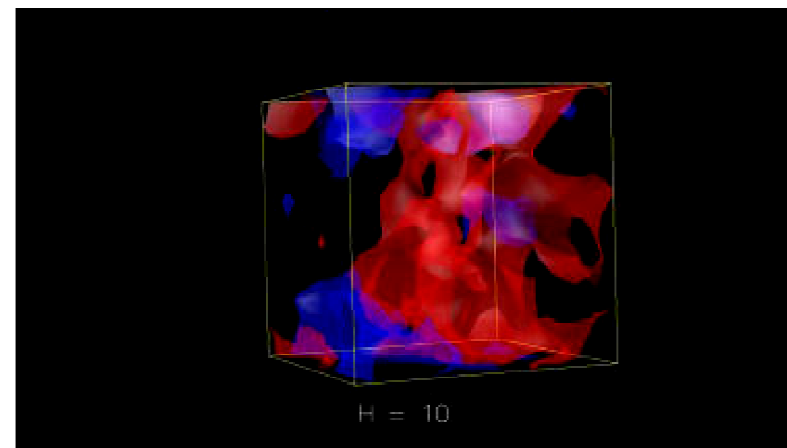
$$B = (500 \text{ MeV})^2$$



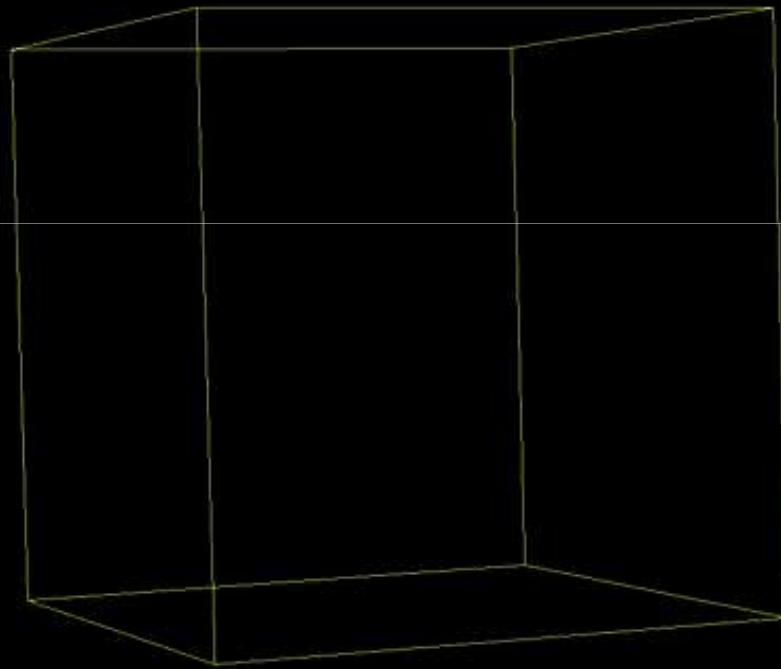
$$B = (780 \text{ MeV})^2$$



$$B = (1.1 \text{ GeV})^2$$



Electric charge density, effect of magnetic field increasing



$$H = 0$$

Magnetic Field Induced Conductivity of the Vacuum

Qualitative definition of conductivity, σ

$$\langle j_\mu(x) j_\nu(y) \rangle = C + A \cdot \frac{\exp\{-m|x-y|\}}{r^\alpha}$$

$$j_\mu(x) = \bar{q}(x) \gamma_\mu q(x)$$

$$\sigma \propto C$$

Magnetic Field Induced Conductivity of the Vacuum

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T} \quad \text{- Conductivity (Kubo formula)}$$

$$G_{ij}(\tau) = \int_0^{+\infty} \frac{dw}{2\pi} K(w, \tau) \rho_{ij}(w),$$

$$K(w, \tau) = \frac{w}{2T} \frac{\cosh\left(w\left(\tau - \frac{1}{2T}\right)\right)}{\sinh\left(\frac{w}{2T}\right)},$$

$$G_{ij}(\tau) = \int d^3\vec{x} \langle j_i(\vec{0}, 0) j_j(\vec{x}, \tau) \rangle$$

Magnetic Field Induced Conductivity of the Vacuum

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T} \quad \text{- Conductivity (Kubo formula)}$$

For weak constant *electric* field

$$\langle \dot{j}_i \rangle = \sigma_{ik} E_k$$

Calculations in SU(2) gluodynamics

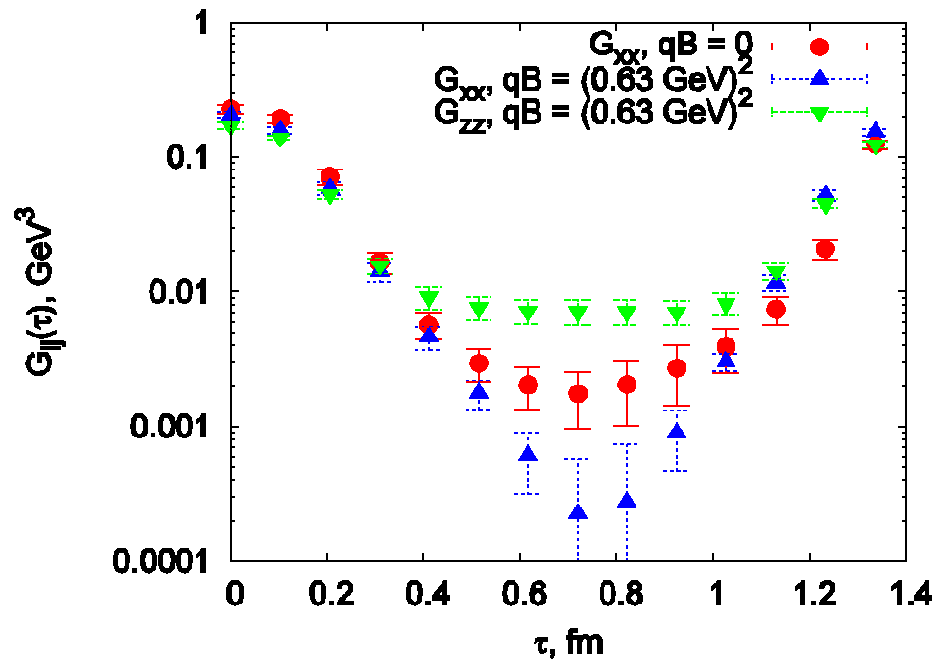
$$\begin{aligned} & \langle \bar{q}(x) \gamma_i q(x) \bar{q}(y) \gamma_j q(y) \rangle \\ &= \int \mathcal{D}A_\mu e^{-S_{YM}[A_\mu]} \text{Tr} \left(\frac{1}{\mathcal{D} + m} \gamma_i \frac{1}{\mathcal{D} + m} \gamma_j \right) \end{aligned}$$

We use overlap operator + Shifted Unitary Minimal Residue Method (*Borici and Allcoci (2006)*) to obtain fermion propagator

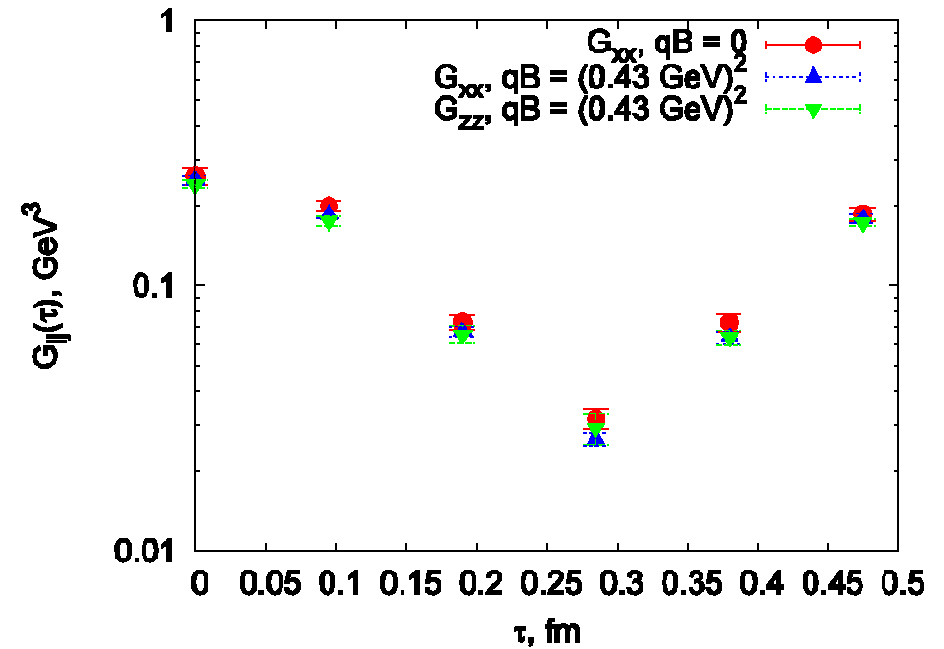
$$G_{ij}(\tau) = \int d^3\vec{x} \langle j_i(\vec{0}, 0) j_j(\vec{x}, \tau) \rangle$$

Calculations in SU(2) gluodynamics

$$G_{ij}(\tau) = \int d^3\vec{x} \langle j_i(\vec{0}, 0) j_j(\vec{x}, \tau) \rangle$$

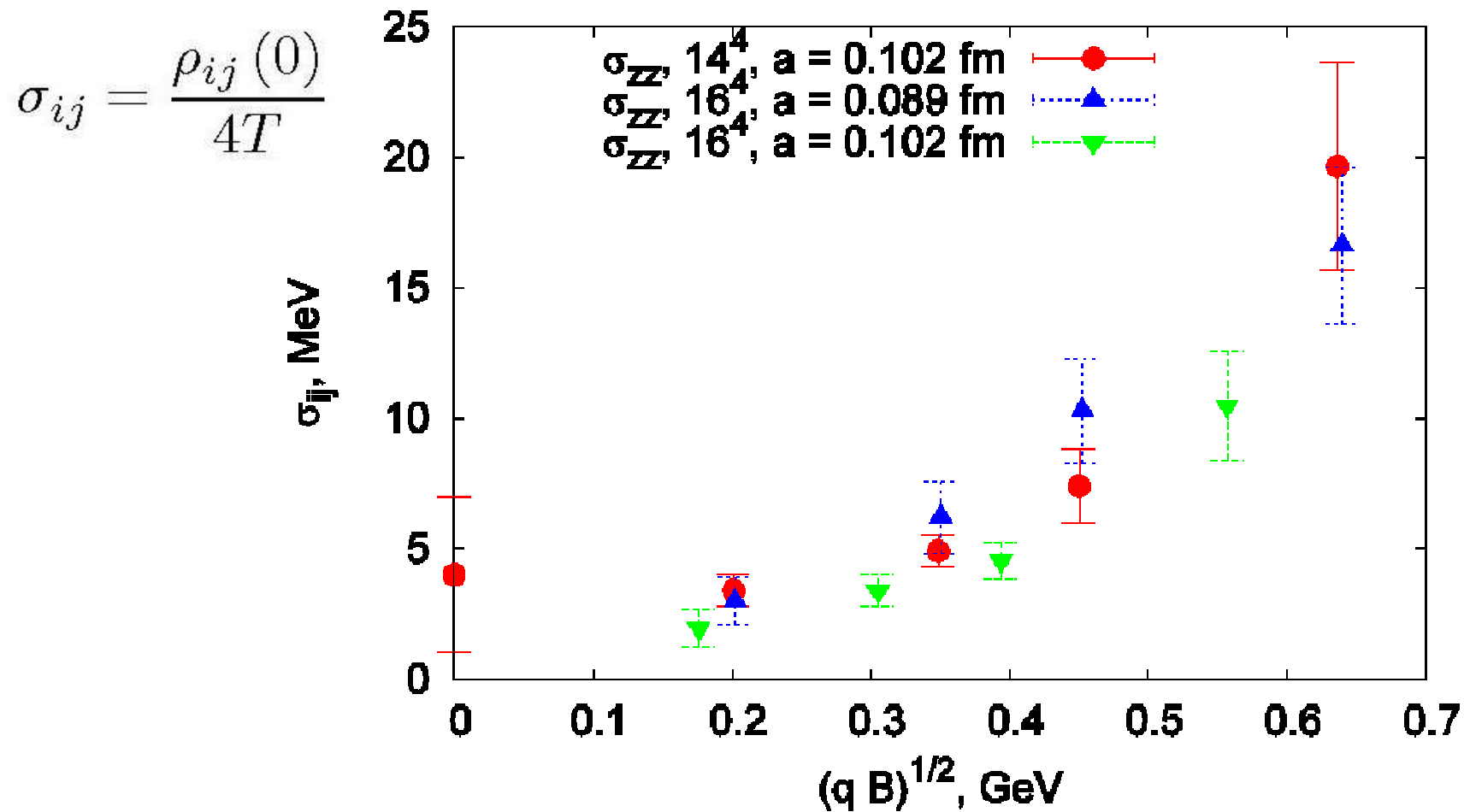


$T = 0$



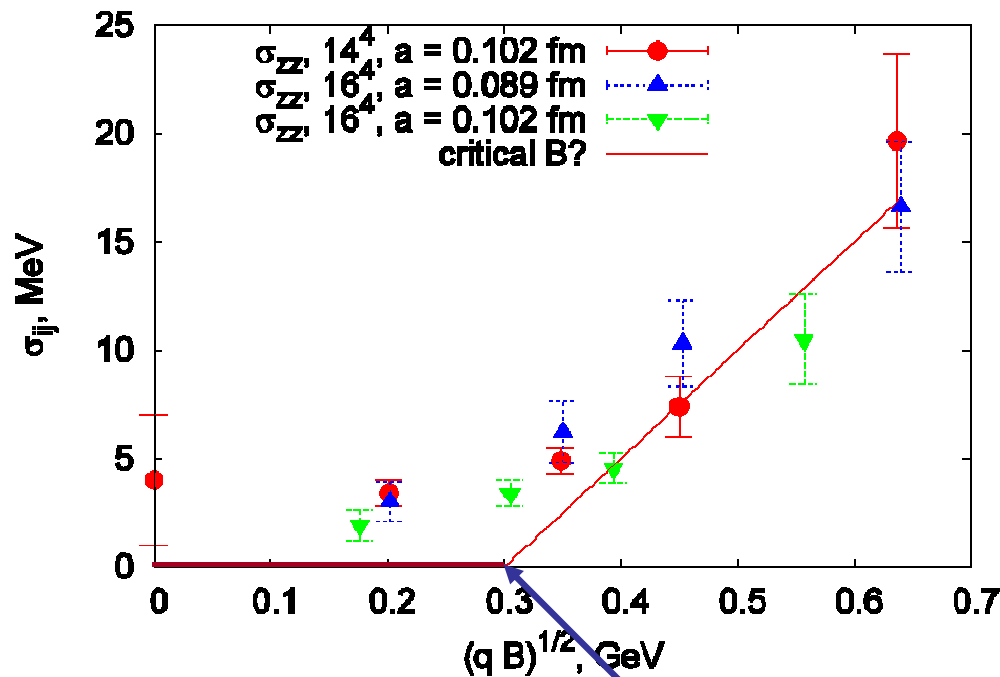
$T/T_c = 1.12$

Calculations in SU(2) gluodynamics, conductivity along magnetic field at $T=0$

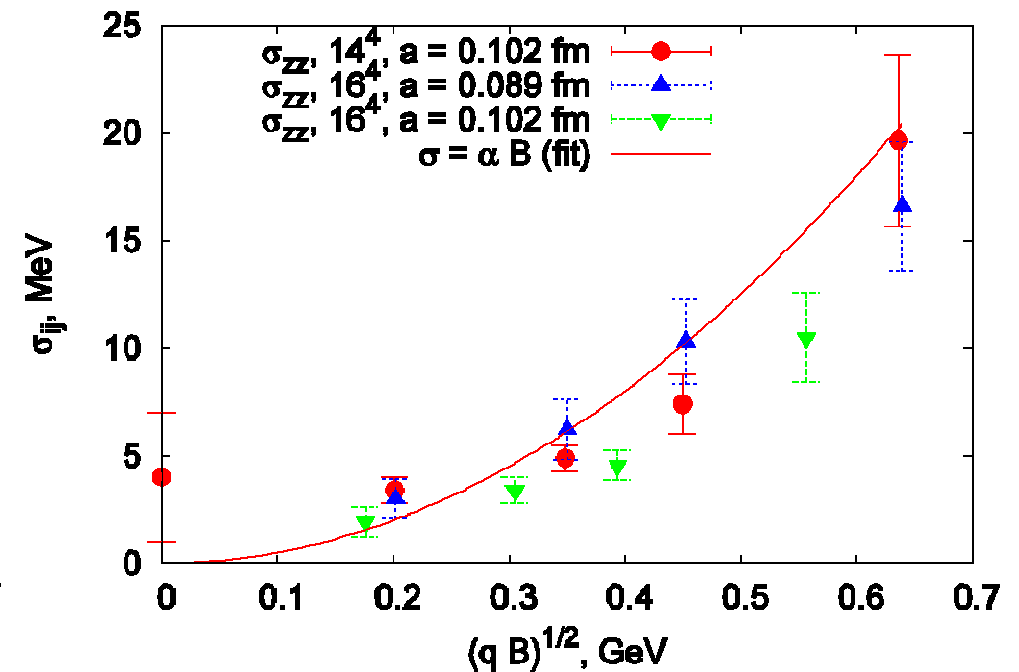


Calculations in SU(2) gluodynamics, conductivity along magnetic field at $T=0$

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

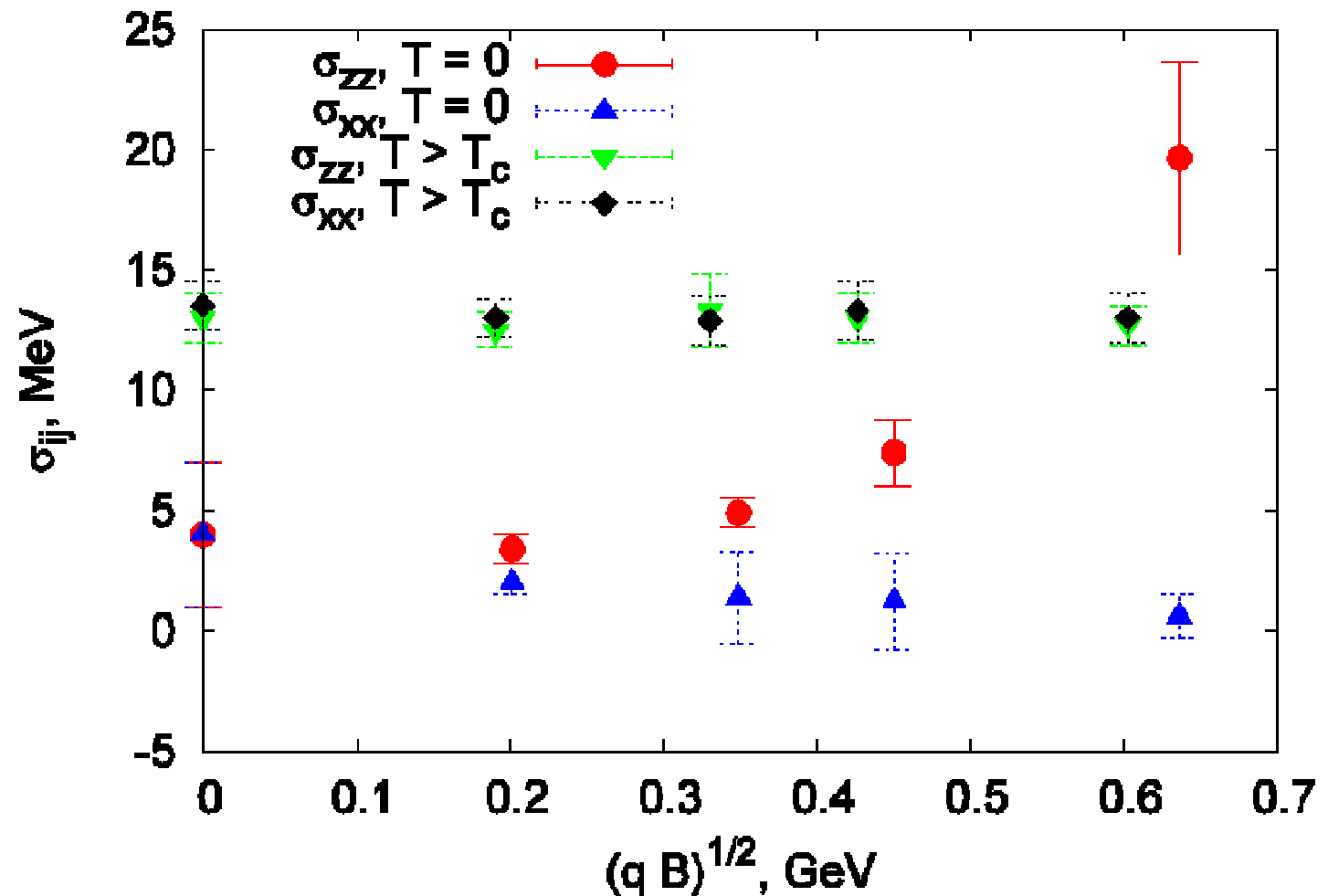


Critical value of magnetic field?

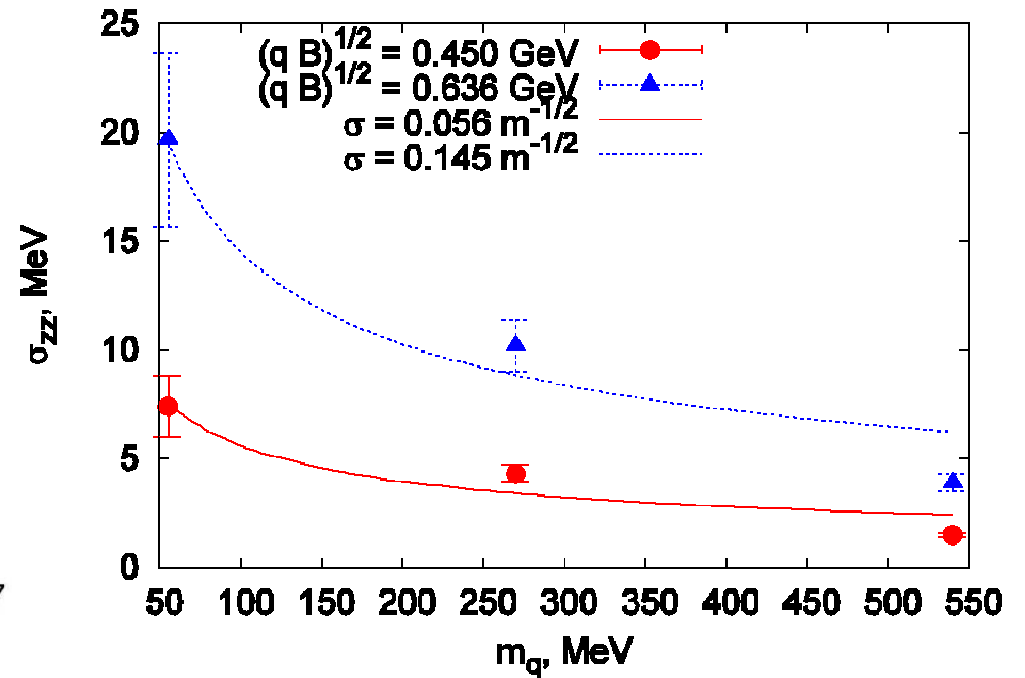
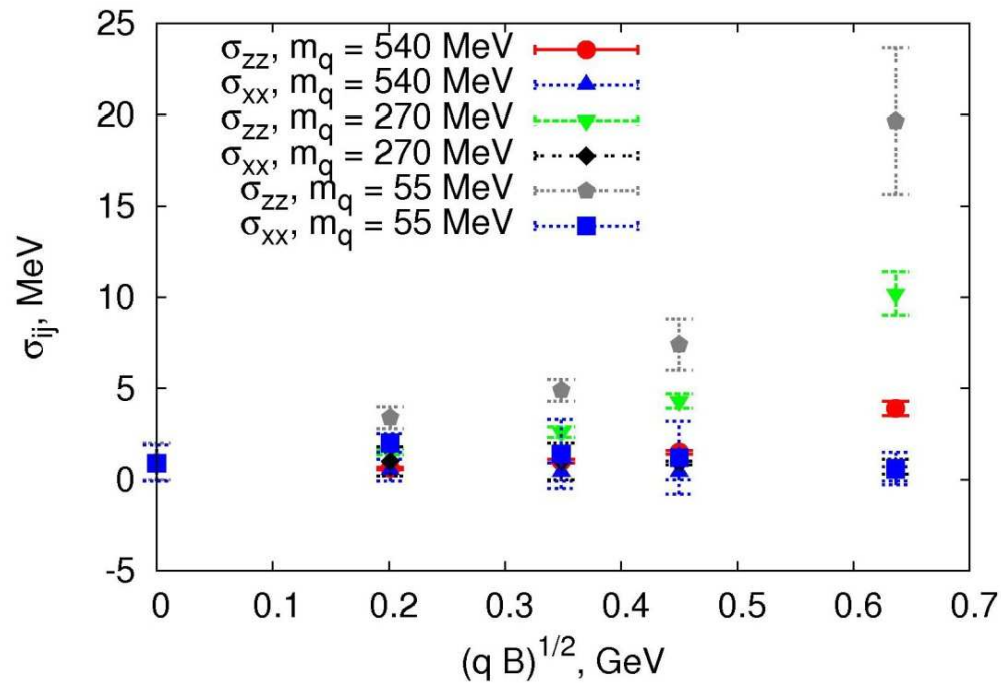


Calculations in SU(2) gluodynamics, conductivity along magnetic field at $T=0, T>0$

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$



Calculations in SU(2) gluodynamics, conductivity along magnetic field at $T=0$, variation of the quark mass

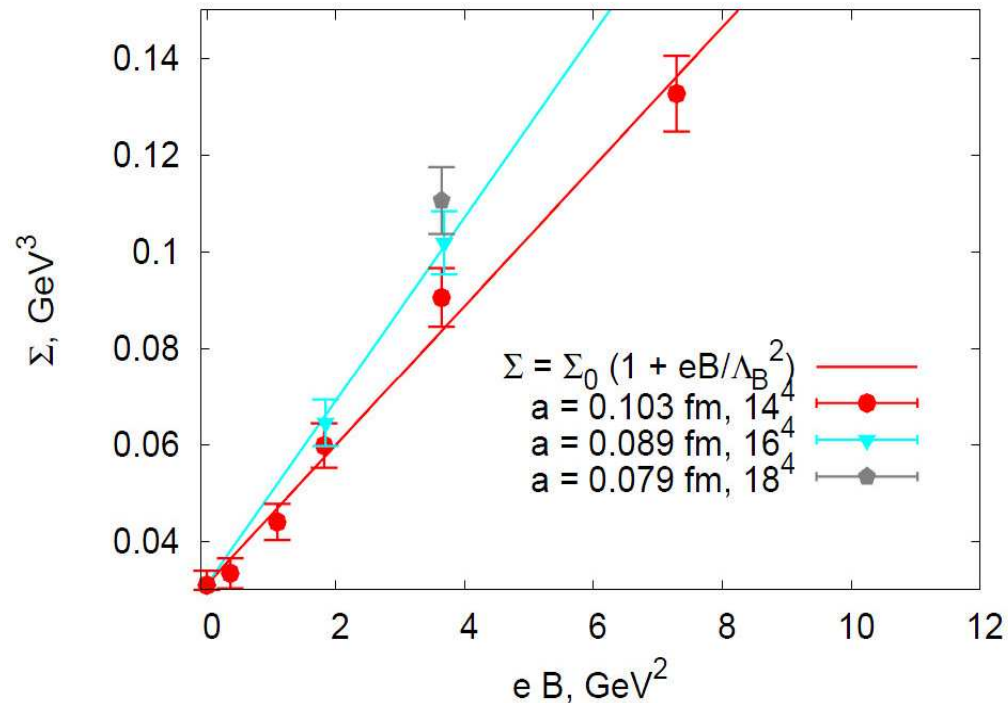


2. Chiral condensate in QCD

$$\Sigma = - \langle \bar{\psi} \psi \rangle$$

$$m_{\pi}^2 f_{\pi}^2 = m_q \langle \bar{\psi} \psi \rangle$$

Chiral condensate vs. field strength, SU(2) gluodynamics



$$\Sigma = \Sigma_0 \left(1 + \frac{eB}{\Lambda_B^2}\right)$$

- Our value for Λ_B :

$$\Lambda_B^{\text{fit}} = (1.41 \pm 0.14 \pm 0.20) \text{ GeV}$$

- χ PT result:

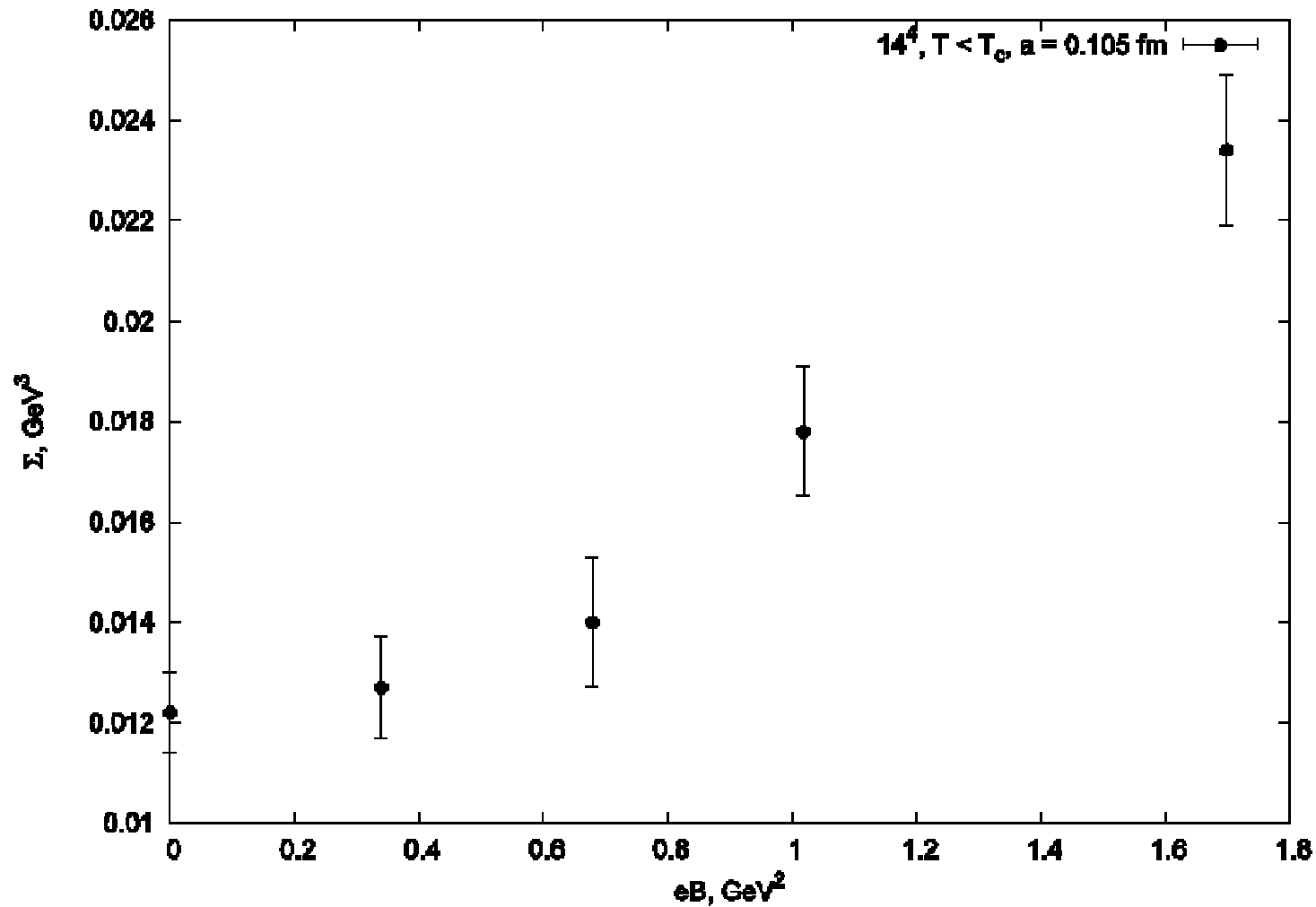
$$\Lambda_B^{\chi PT} = 1.96 \text{ GeV} \quad (F_\pi = 130 \text{ MeV} - \text{real world})$$

$$\Lambda_B^{\chi PT} = 1.36 \text{ GeV} \quad (F_\pi = 90 \text{ MeV} - \text{quenched})$$

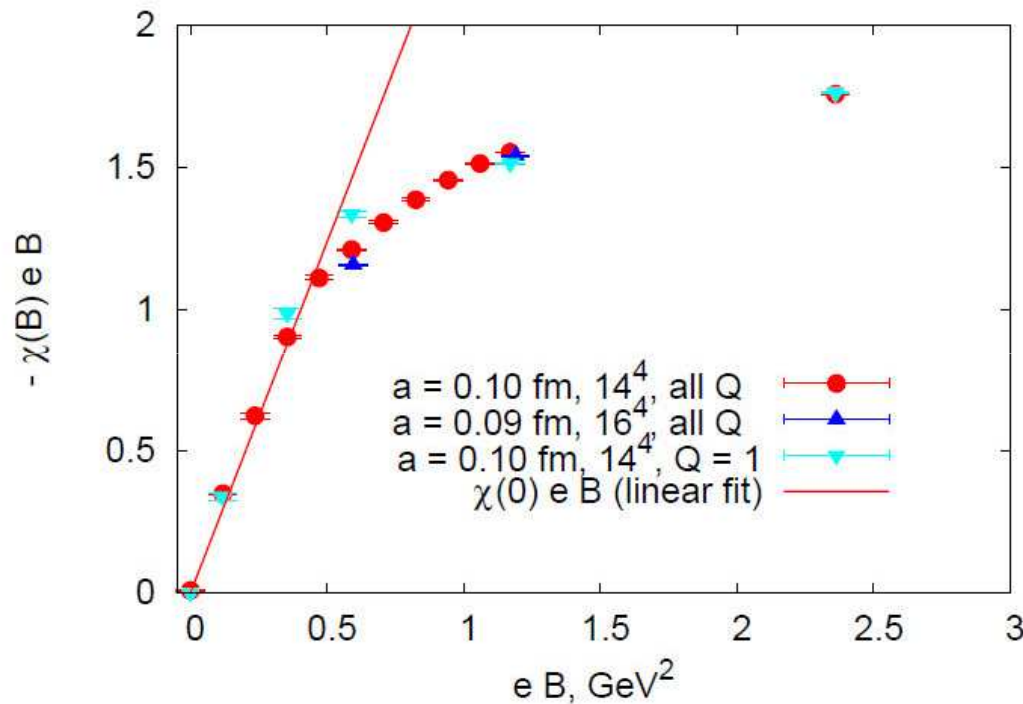
- Chiral condensate at $B = 0$: $\Sigma_0^{\text{fit}} = [(310 \pm 6) \text{ MeV}]^3$

We are in agreement with the chiral perturbation theory: the chiral condensate is a linear function of the strength of the magnetic field!

Chiral condensate vs. field strength, SU(3) gluodynamics



3. Magnetization of the vacuum as a function of the magnetic field



Spins of virtual quarks turn parallel to the magnetic field



$$\langle \bar{\psi} \sigma_{\alpha\beta} \psi \rangle = \chi \langle \bar{\psi} \psi \rangle F_{\alpha\beta}$$

$$\sigma_{\alpha\beta} = \frac{1}{2i} [\gamma_{\alpha}, \gamma_{\beta}]$$

$$\langle \bar{\psi} \psi \rangle \chi = -46(3) \text{ MeV} \leftrightarrow \text{our result}$$

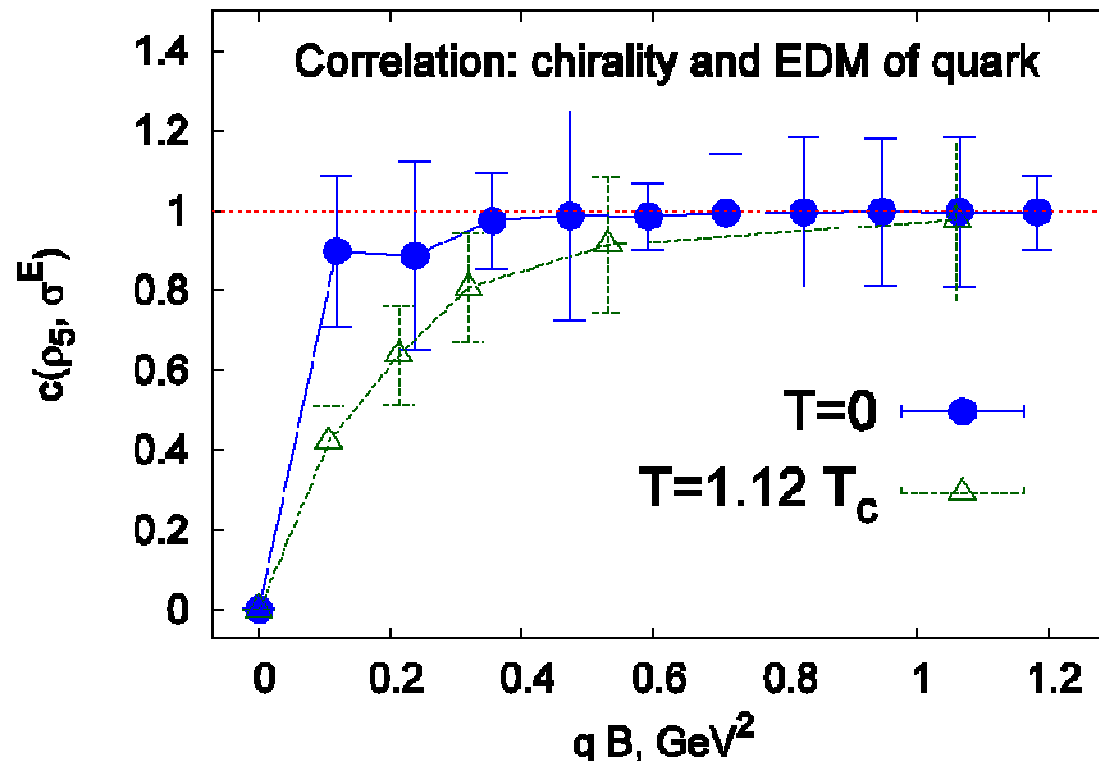
$$\langle \bar{\psi} \psi \rangle \chi \approx -50 \text{ MeV} \leftrightarrow \text{QCD sum rules}$$

(I. I. Balitsky, 1985, P. Ball, 2003.)

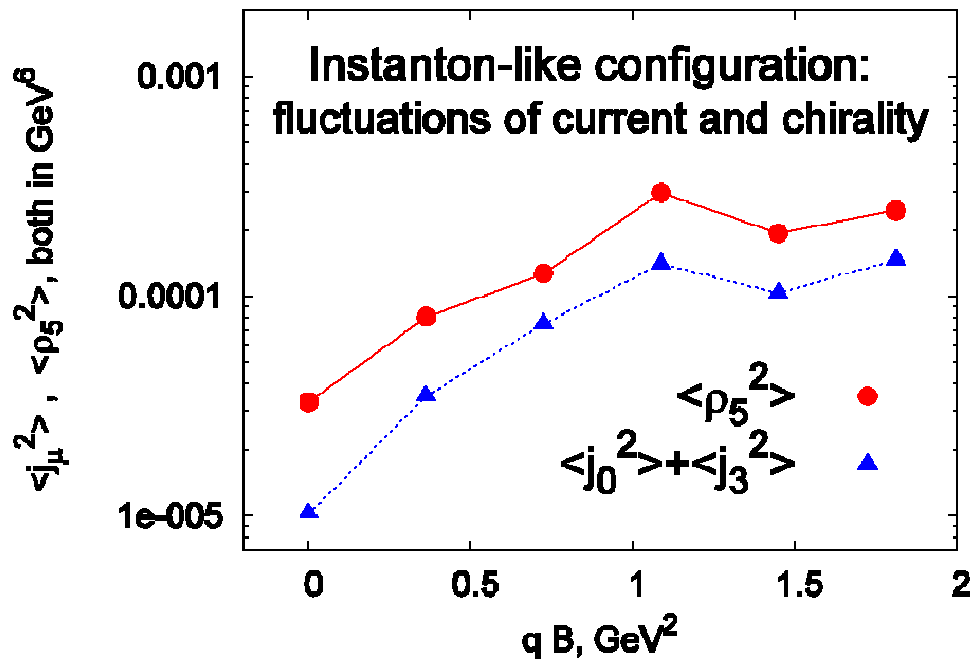
4. Generation of the anomalous quark electric dipole moment along the axis of magnetic field

Large correlation between square of the electric dipole moment

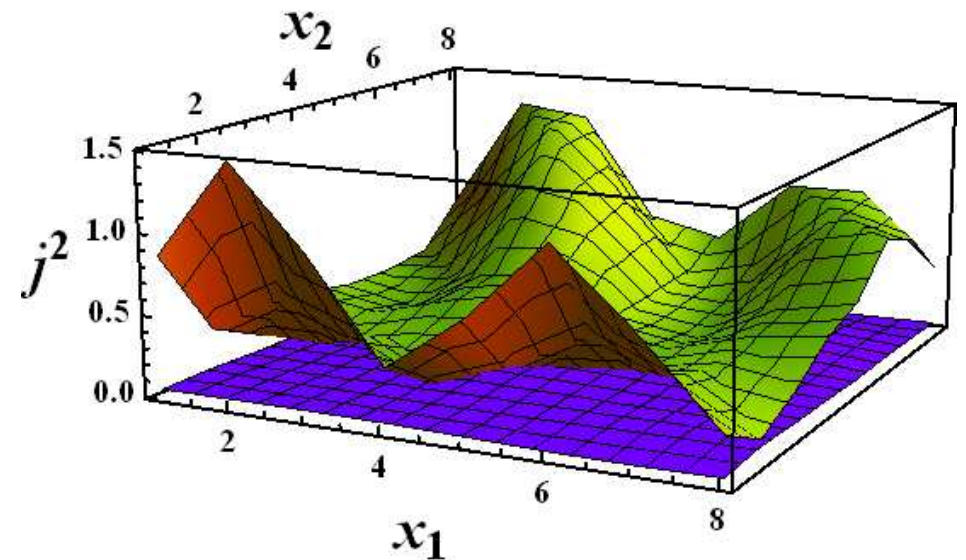
$$\sigma_{0i} = i\bar{\psi}[\gamma_0, \gamma_i]\psi \quad \text{and chirality} \quad \rho_5 = \bar{\psi}\gamma_5\psi$$



5. Electric currents in instanton field+magnetic field (CME)



The fluctuations of the chirality $\rho_5 = \bar{\psi} \gamma_5 \psi$ and the fluctuations of the longitudinal electric current as a function of the magnetic field.



The squared components of the electric current in a 12-plane. The upper sheet represents the spatial distribution of the longitudinal current, the lower sheet corresponds to the transverse current.

$$\langle j_i^2 \rangle_{IR} = \langle j_i^2(H, T) \rangle - \langle j_i^2(0, 0) \rangle, \quad j_i = \bar{\psi} \gamma_i \psi$$

Conclusions

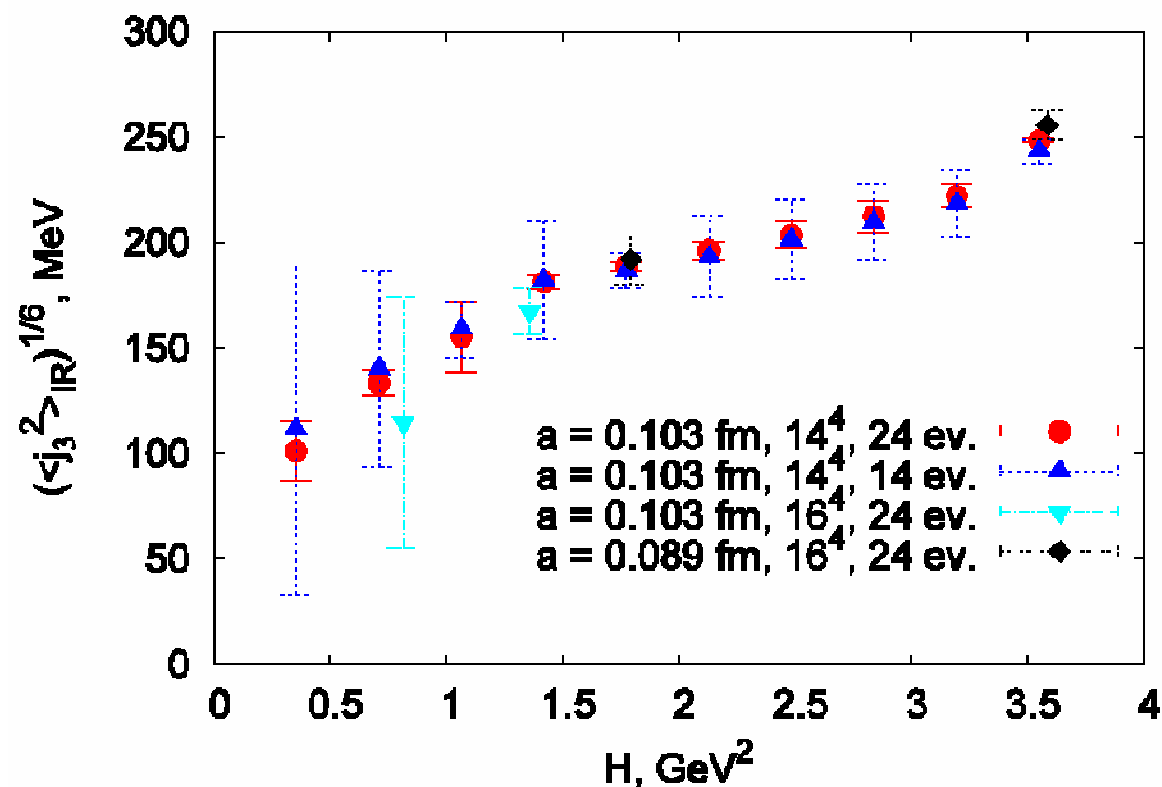
- 1. We observe that in the confinement phase the external magnetic field induces nonzero electric conductivity along the direction of the magnetic field, transforming the system from an insulator into an anisotropic conductor ([arXiv:1003.2180](#)).
- 2. In the deconfinement phase the conductivity does not exhibit any sizable dependence on the magnetic field ([arXiv:1003.2180](#)).
- 3. The conductivity is weaker for heavy quarks, thus it is interesting to measure experimentally the charge asymmetry for S and C quarks.

Conclusions

- 4. We observe that the chiral condensate is proportional to the strength of the magnetic field, the coefficient of the proportionality agrees with Chiral Perturbation Theory. Microscopic mechanism for the chiral enhancement is the localization of fermion modes in the vacuum (arXiv:0812.1740, Phys.Lett. B 682:484-489,2010).
- 5. The calculated vacuum magnetization is in a qualitative agreement with model calculations (arXiv:0906.0488, Nucl.Phys. B 826 (2010) 313).
- 6. We observe very large correlation between electric dipole moment of quark and chirality (arXiv:0909.2350 Phys.Rev.D81:036007,2010).

Appendix

Chiral Magnetic Effect on the lattice, numerical results **nearzero**



Regularized electric current:

$$\langle j_3^2 \rangle_{IR} = \langle j_3^2(H, T) \rangle - \langle j_3^2(0, 0) \rangle, \quad j_3 = \bar{\psi} \gamma_3 \psi$$

Chiral Magnetic Effect on the lattice, numerical results

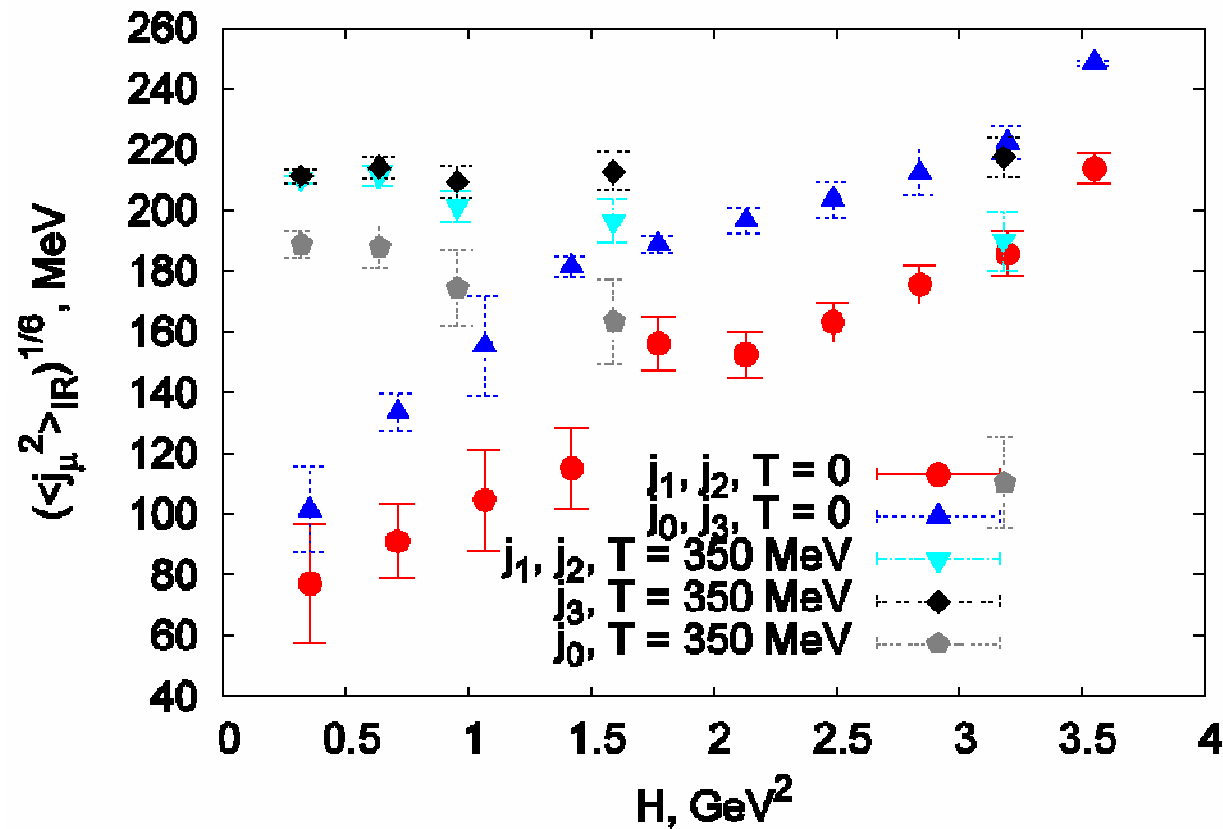
near T_c and near zero

$$T=0$$

$$F_{12} \neq 0$$

$$\langle j_1^2 \rangle = \langle j_2^2 \rangle$$

$$\langle j_3^2 \rangle = \langle j_0^2 \rangle$$



$$T>0$$

$$F_{12} \neq 0$$

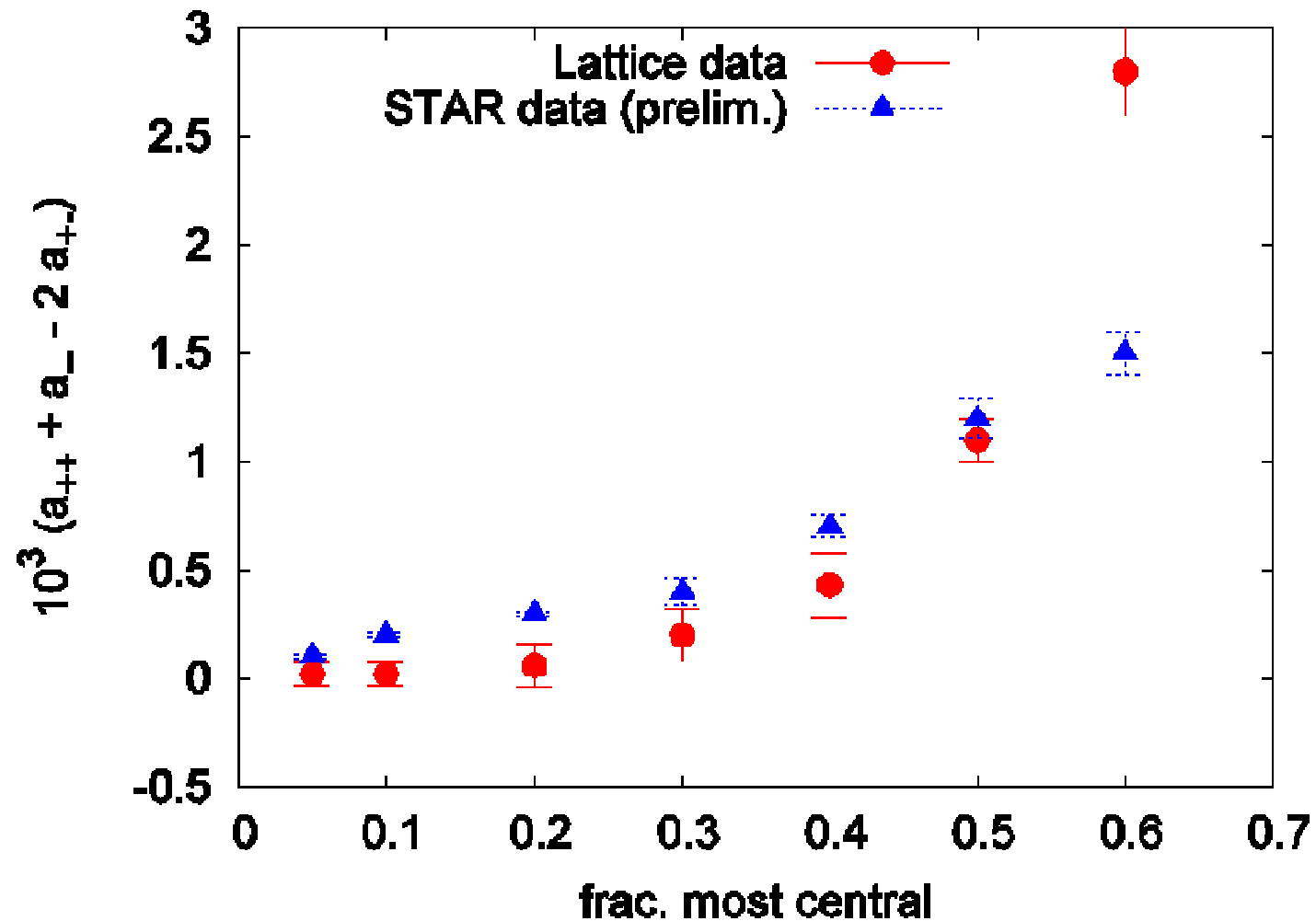
$$\langle j_1^2 \rangle = \langle j_2^2 \rangle$$

$$\langle j_3^2 \rangle \neq \langle j_0^2 \rangle$$

Regularized electric current:

$$\langle j_i^2 \rangle_{IR} = \langle j_i^2(H, T) \rangle - \langle j_i^2(0, 0) \rangle, \quad j_i = \bar{\psi} \gamma_i \psi$$

Chiral Magnetic Effect, EXPERIMENT VS LATTICE DATA (Au+Au)



Chiral Magnetic Effect, EXPERIMENT VS LATTICE DATA

$$a_{ab} = \frac{1}{N_e} \sum_{e=1}^{N_e} \frac{1}{N_a N_b} \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} \cos(\phi_{ia} + \phi_{jb})$$

$$\frac{\langle (\Delta Q)^2 \rangle}{N_q^2} = a_{++} + a_{--} - 2a_{+-}$$

experiment

$$\begin{aligned} R &\approx 5 \text{ fm} \\ \rho &\approx 0.2 \text{ fm} \\ \tau &\approx 1 \text{ fm} \end{aligned}$$

our fit

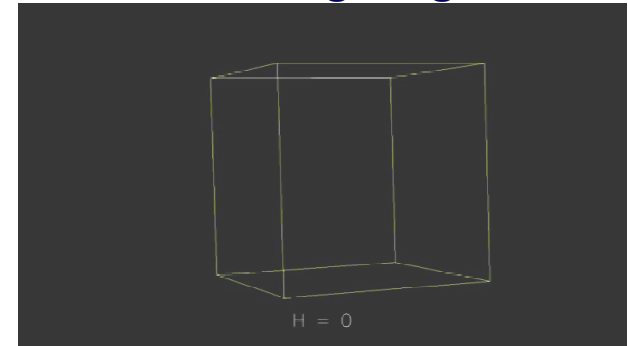
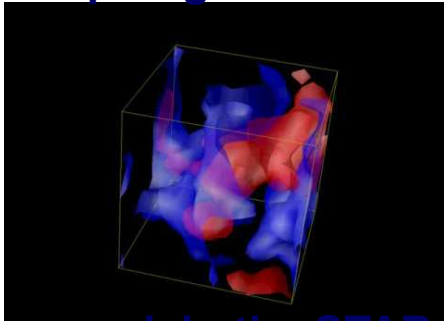
D. E. Kharzeev,
L. D. McLerran, and
H. J. Warringa,
Nucl. Phys. A 803,
227 (2008),

$$= \frac{4\pi \tau^2 \rho^2 R^2}{3N_q^2} \left(\langle j_{\parallel}^2 \rangle + 2\langle j_{\perp}^2 \rangle \right)$$

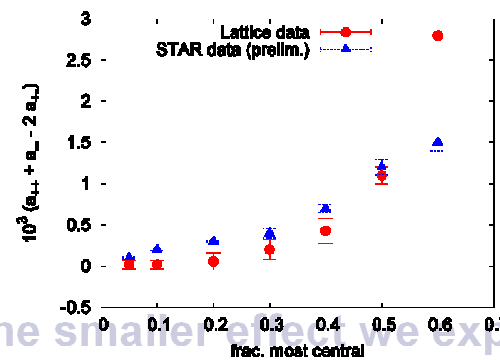
our lattice data at $T=350 \text{ MeV}$

Lessons from computer calculations

1. In the vacuum of QCD we observe the **charge separation** in the strong magnetic field, the topological structure is complicated



2. We can explain the STAR experimental data, but **the growth of asymmetry is due to the kinematical factor**, and is not related to the growth of the magnetic field



3. The larger is the quark mass **the smaller effect we expect**, thus it is important to **measure the asymmetry for mesons containing S and C quarks**

Results [arXiv:0909.1808](#)

- 1. We observe signatures of the Chiral Magnetic Effect, but the physics may differ from the model of Kharzeev, McLerran and Warringa ([arXiv:0907.0494](#), [Phys.Rev.D79:106003,2009](#))
- 2. We observe that the chiral condensate is proportional to the strength of the magnetic field, the coefficient of the proportionality agrees with Chiral Perturbation Theory. Microscopic mechanism for the chiral enhancement is the localization of fermion modes in the vacuum ([arXiv:0812.1740](#), [Phys.Lett. B682\(2010\)484](#))
- 3. The calculated vacuum magnetization is in a qualitative agreement with model calculations ([arXiv:0906.0488](#), [Nucl.Phys. B 826 \(2010\) 313](#))
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